Consider the polar equation $r = 2 + 2\sin 2\theta$.

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POLAR AXIS POLE POLAR AXIS

The following symmetry tests all fail: $(r, -\theta)$, $(-r, \theta)$ and $(-r, \pi - \theta)$

[a] Is the graph symmetric with respect to the polar axis ? **State your conclusion clearly.**

(1) NO CONCLUSION, (BOTH TESTS FAIL)

[b] Is the graph symmetric with respect to $\theta = \frac{\pi}{2}$? **State your conclusion clearly.** NO CONCLUSION (1)

$(r, -\theta)$ $-r = 2 + 2\sin 2(-\theta)$ (½)

$-r = 2 - 2\sin 2\theta$

$r = -2 + 2\sin 2\theta$ (½)

$(r, \pi - \theta)$ $r = 2 + 2\sin 2(\pi - \theta)$ (½)

$r = 2 + 2\sin(2\pi - 2\theta)$

$r = 2 + 2[\sin 2\pi \cos 2\theta - \cos 2\pi \sin 2\theta]$

[c] Is the graph symmetric with respect to the pole ? **State your conclusion clearly.** $r = 2 - 2\sin 2\theta$, (1)

$(r, \pi + \theta)$ $r = 2 + 2\sin 2(\pi + \theta)$ (½)

$r = 2 + 2\sin(2\pi + 2\theta)$

$r = 2 + 2[\sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta]$

(1) $r = 2 + 2\sin 2\theta$, SYMMETRIC (1)

[d] Based on the symmetry tests, what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in [0, \pi]$ OR $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ (1) FOR EITHER INTERVAL

[e] Find the zeros of the graph in the minimum interval from [d]
(ie. for what values of θ in the minimum interval does the graph pass through the pole ?).

$0 = 2 + 2\sin 2\theta$

(½) $\sin 2\theta = -1$ (½) FOR EITHER 2θ

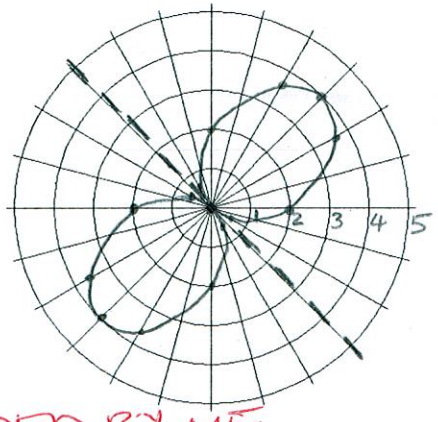
$2\theta = \frac{3\pi}{2}$ OR $-\frac{\pi}{2}$

$\theta = \frac{3\pi}{4}$ OR $-\frac{\pi}{4}$ (1) FOR EITHER θ

[f] Find the value of r for all the common values of θ in the minimum interval. Plot those points. Connect the points into a curve. Reflect that part of the curve using the results of the symmetry tests in [a], [b] and [c] to draw the complete graph.

CALCULATE THE *r* –VALUES ON SCRATCH PAPER ON THE BACK. WRITE THE POLAR COORDINATES HERE.

$(2, 0)$
$(2 - \sqrt{3}, -\frac{\pi}{6})$ $(2 + \sqrt{3}, \frac{\pi}{6})$ $(2 - \sqrt{3}, \frac{2\pi}{3})$
$(0, -\frac{\pi}{4})$ $(4, \frac{\pi}{4})$ $(0, \frac{3\pi}{4})$
$(2 - \sqrt{3}, -\frac{\pi}{3})$ $(2 + \sqrt{3}, \frac{\pi}{3})$ $(2 - \sqrt{3}, \frac{5\pi}{6})$
$(2, -\frac{\pi}{2})$ $(2, \frac{\pi}{2})$ $(2, \pi)$



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Consider the conic with polar equation $r = \dfrac{20}{3 - 7\sin\theta}$.

SCORE: ____ / 14 PTS

[a] What is the type of the conic ? **Justify your answer clearly.**

$r = \dfrac{\frac{20}{3}}{1 - \frac{7}{3}\sin\theta}$ $e = \frac{7}{3} > 1$ (1) HYPERBOLA (1)

[b] What is the equation of the directrix ?

$ep = \frac{20}{3}$

$\frac{7}{3}p = \frac{20}{3} \rightarrow p = \frac{20}{7}$ (1) $y = -\frac{20}{7}$ (1) (½)

[c] Find the **polar AND rectangular** coordinates of the $x-$ and $y-$ intercepts.

θ	r
0	$\frac{20}{3}$
$\frac{\pi}{2}$	-5
π	$\frac{20}{3}$
$\frac{3\pi}{2}$	2

X-INT POLAR $(\frac{20}{3}, 0)$ $(\frac{20}{3}, \pi)$ (½)

RECT $(\pm\frac{20}{3}, 0)$ (½)

y-INT POLAR $(-5, \frac{\pi}{2})$ $(2, \frac{3\pi}{2})$ (½)

RECT $(0, -5)$ $(0, -2)$ (½)

[d] What are the rectangular coordinates of the vertices, center, foci, and endpoints of the latera recta ?

VERTICES:

$(0, -5)$ $(0, -2)$ (1)

CENTER:

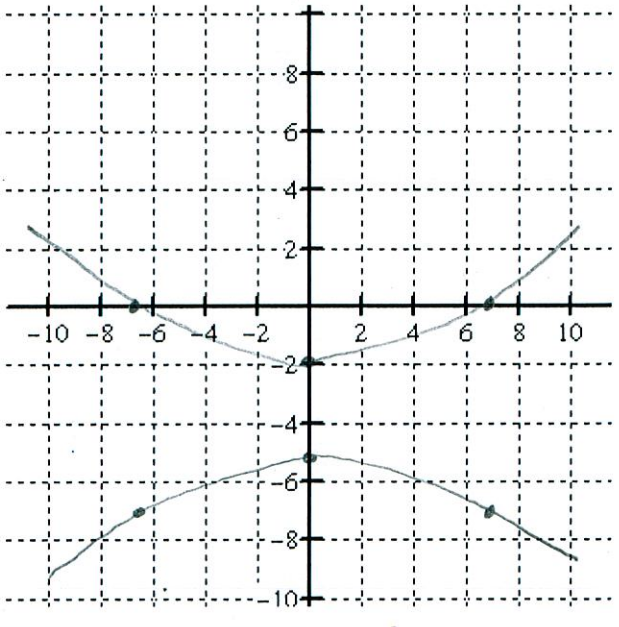
$(0, \frac{-5-2}{2}) = (0, -\frac{7}{2})$ (1)

FOCI:

$(0, 2 \cdot -\frac{7}{2}) = (0, -7)$ (1) AND $(0, 0)$ (½)

ENDPOINTS OF LATERA RECTA:

$(\pm\frac{20}{3}, 0)$ (1) $(\pm\frac{20}{3}, -7)$ (1)



[e] Graph the conic by connecting the relevant points from [d] appropriately.

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